ARJUNA

JEE

LECTURE NUMBER - 8

Basic Math

MSM SIR

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TABLE OF CONTENT

01 DPP DOUBTS DISCUSSION (POPULAR DOUBTS)

02 MODULUS INEQUALITIES

DPP DOUBTS (POPULAR)

DPP3

1. Exhaustive set of values of 'x' satisfying:

$$\left(\frac{x-x^2}{x^2+2x}\right)\geq 0$$

(1) $x\in(-2,1]$

(2) $x\in[-2,-1]$

(3) $x\in(-2,1]-\{0\}$

(4) $x\in(-\infty,-2]\cup[1,\infty)$

$$\frac{x(1-x)}{x(x+2)}\geq 0$$

$x\neq 0$

$$\frac{1-x}{x+2}\geq 0$$

$$\frac{-(1-x)}{x+2}\leq 0$$

$$\frac{x-1}{x+2}\leq 0$$

$x\in(-2,1]-\{0\}$

DPP3

2. Complete set of values of 'x' satisfying:

$$\frac{(x-1)^{15}x^{28}}{(x+1)^{2021}(x+2)^{2020}} \geq 0$$

(1) $x \in (-\infty, -1) \cup [1, \infty)$

(2) $x \in (-1, 1] - \{0\}$

(3) $x \in (-\infty, -2] \cup (0, \infty)$

(4) $x \in (-\infty, -2) \cup (-2, -1) \cup [1, \infty) \cup \{0\}$ ✓

$$\frac{(x-1)}{x+1} \geq 0$$

$x = 0$

$x \neq -2$

$x \in (-\infty, -1) \cup [1, \infty) - \{-2\} \cup \{0\}$

OR,

$x \in (-\infty, -2) \cup (-2, -1) \cup [1, \infty) \cup \{0\}$

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DPP3

6. $x^4 - 2x^2 - 63 \leq 0$

$x^2 = t$

$t^2 - 2t - 63 \leq 0$

$(t-9)(t+7) \leq 0$

$t = x^2$

$(x^2-9)(x^2+7) \leq 0$

$(x-3)(x+3)(x^2+7) \leq 0$

- $x^2 + 0 \cdot x + 7$

$\cap \begin{cases} a = 1 \\ D = (0)^2 - 28 < 0 \end{cases}$

AP, CB

$(x-3)(x+3) \leq 0$

$x \in [-3, 3]$ Ans.

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DPP3

3. Solution of inequality: $\frac{x-2}{x^2+1} \le 0$

(1) $x \in [2, \infty)$

(2) $x \in (-\infty, -1) \cup (1, 2]$

(3) $x \in (-\infty, 2]$

(4) $x \in (-1, 1) \cup [2, \infty)$

$\frac{x-2}{x^2+1} \le 0$

$x^2 + 0 \cdot x + 1$

$a = 1 \quad (a > 0)$

$D = -4 \quad (D < 0)$

AP, CB

$(x-2) \le 0$

$x \in (-\infty, 2]$

Ans

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DPP2 (PATA NAHI KI KYUN POOCHAA !)

10. Solution of $(x-1)^2(x+1)^3(x-4) \geq 0$ is

(1) $x \in (-\infty, -1] \cup \{2\} \cup [4, \infty)$

(2) $x \in (-\infty, 1] \cup \{3\} \cup [4, \infty)$

(3) $x \in (-\infty, -1] \cup \{1\} \cup [4, \infty)$

(4) $x \in (-\infty, -1] \cup \{1\} \cup [3, \infty)$

$x = 1$

$\longrightarrow (x+1)(x-4) \geq 0$

$x \in (-\infty, -1] \cup [4, \infty) \cup \{1\}$

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HOMEWORK DISCUSSION

Q. Solving the inequality

$x^4 + 6x^3 + 6x^2 + 6x + 5 < 0$ (CHALLENGE - *a guru*)

$x^4+6x^3+6x^2+6x+6-1<0$

$(x^4-1)+6x^2(x+1)+6(x+1)<0$

$(x-1)(x+1)(x^2+1)+6x^2(x+1)+6(x+1)<0$

$(x+1)\left[(x-1)(x^2+1)+6(x^2+1)\right]<0$

$(x+1)(x^2+1)\left[x-1+6\right]<0$

(x^4-1)
$=(x^2)^2-(1)^2$
$=(x^2-1)(x^2+1)$
$=(x-1)(x+1)(x^2+1)$

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$$(x+1)(x+5)\underbrace{(x^2+1)}_{\text{AP}_0,\text{CB}} < 0$$

$$(x+1)(x+5) < 0$$

$x \in (-5, -1)$ Ans

Chote

Bada

x^2+1

$= x^2+0.x+1$

$a=1$ } $a>0$

$D=-4$ } $D<0$

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RECAP

BABA KA GYAAN:

If $|x-a|+|x-b|=c$, then

① $c=|a-b|$; $x\in[a,b]$

② $c>|a-b|$; $x\in\{a-d, b+d\}$; $d=\frac{c-|a-b|}{2}$

③ $c<|a-b|$; $x\in\phi$

(LEARN)

Q. $|x-2|+|x-4| \geq 6$, *set of exhaustive values of* x ? (HW)

कल wallah Process

$a=2 \quad c=6$

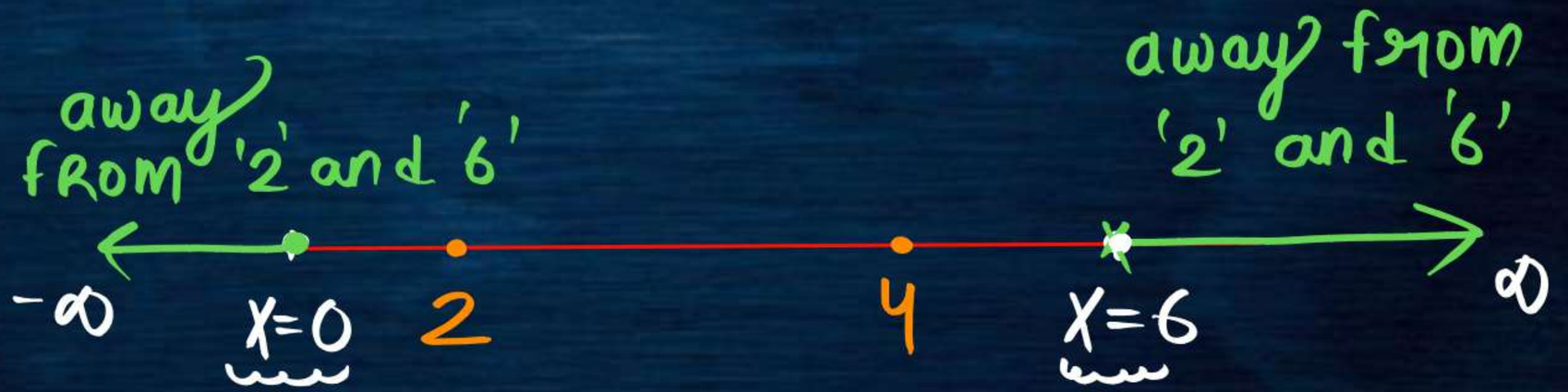
$b=4$

$|x-2|+|x-4|=6 \longrightarrow x \in \{0,6\}$

$|a-b|=2$

$c>|a-b|$

$d=\frac{6-2}{2}=2$



$$x \in (-\infty, 0] \cup [6, \infty)$$

Ans.

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Q → Find 'x' satisfying $|x-7|+|x-9| \geq 8$.

Sol^n →

$|x-7|+|x-9| = 8$

$a = 7, \; b = 9$

$c = 8$

$|a-b| = 2$

$c > |a-b|$

$d = \frac{8-2}{2} = 3$

$x \in [4, 12]$

$x \in (-\infty, 4] \cup [12, \infty)$ Ans

Q → Find 'x' satisfying $|x-7|+|x-9|>6$.

$|x-7|+|x-9|=6$

$a=7 \quad b=9$

$|a-b|=2$

$c=6$

$c>|a-b|$

$d=c-\frac{|a-b|}{2}$

$=\frac{6-2}{2}=2$

$x \in \{5, 11\}$

$x \in (-\infty, 5) \cup (11, \infty)$ Ans

MODULUS INEQUALITIES

TYPE 1

$|x| \leq a$; $a \rightarrow$ Positive

$\Rightarrow x \in [-a, a]$

OR

$-a \leq x \leq a$

PROOF $\rightarrow$ $|x| \leq a \xRightarrow{\text{sq.}} |x|^2 \leq a^2$

$x^2 - a^2 \leq 0$

$(x-a)(x+a) \leq 0$

$x \in [-a, a]$

LEARN

$\Rightarrow |x| < a$; $a \rightarrow$ Positive

$x \in (-a, a)$

OR

$-a < x < a$

LEARN

PROOF $\rightarrow$ $|x| < a \xRightarrow{\text{squaring}} |x|^2 < a^2$

$x^2 - a^2 < 0$

$(x-a)(x+a) < 0$

$x \in (-a, a)$

Generalise, →

① $|E| \leq a$

$-a \leq E \leq a$

② $|E| < a$

$-a < E < a$

$a \rightarrow$ Positive

Q. Solve for x :

$$|x - 5| \leq 2.$$

$E = x - 5$

$|E| \leq 2$

$\Rightarrow -2 \leq E \leq 2$

$-2 \leq x - 5 \leq 2$

$5 - 2 \leq x \leq 2 + 5$

$3 \leq x \leq 7$

$x \in [3, 7]$

Ans.

Q. Solve for x :

$$|x + 2| < 0.$$

Not possible

$x \in \phi$ Ans

Modulus can never be lesser than zero

Gyaan

Q. Solve for x :

$$\left|\frac{3x+3}{5}\right| < 3.$$

$E = \frac{3x+3}{5}$

$|E| < 3$

$-3 < E < 3$

$-3 < \frac{3x+3}{5} < 3$

$-15 < 3x+3 < 15$

$-18 < 3x < 12$

$-6 < x < 4$

OR

$x \in (-6, 4)$ Ans

Q. Solve for x :

$$\left|\frac{x^2-3x-1}{x^2+x+1}\right| < 3.$$

start

$$E = \frac{x^2-3x-1}{x^2+x+1}$$

$$|E| < 3$$

$$-3 < E < 3$$

DOUBLE INEQUALITY

$$-3 < \frac{x^2-3x-1}{x^2+x+1} < 3$$

$$\cap \begin{cases} -3 < \dfrac{x^2-3x-1}{x^2+x+1} \\ \dfrac{x^2-3x-1}{x^2+x+1} < 3 \end{cases}$$

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$$-3 < \frac{x^2-3x-1}{x^2+x+1}$$

$$\Rightarrow \frac{x^2-3x-1}{x^2+x+1} + 3 > 0$$

$$\frac{x^2-3x-1+3x^2+3x+3}{x^2+x+1} > 0$$

CB, APo

$$\frac{(4x^2+2)\cdot 1}{x^2+x+1} > 0$$

APo, CB

$$\Rightarrow \boxed{1 > 0}$$

(always TRUE

$$\boxed{x \in (-\infty, \infty)}$$

(α)

$$\frac{x^2-3x-1}{x^2+x+1}-3<0$$

$$\frac{x^2-3x-1-3x^2-3x-3}{x^2+x+1}<0$$

$$\frac{-2x^2-6x-4}{x^2+x+1}<0$$

$$\frac{2x^2+6x+4}{x^2+x+1}>0$$

x^2+x+1

AP0, CB

$2(x^2+3x+2)>0$

$2(x+1)(x+2)>0$

$x\in(-\infty,-2)\cup(-1,\infty)$

B

$q \cap \beta$

$x \in (-\infty, -2) \cup (-1, \infty)$ Ans.

TYPE 2

$|x| \geq a$ $\quad a \to$ Positive

$x \in (-\infty, -a] \cup [a, \infty)$

OR

$x \leq -a \quad \cup \quad x \geq a$

PROOF →

$|x| \geq a$

$\Rightarrow |x|^2 \geq a^2$

$(x^2 - a^2) \geq 0$

$(x-a)(x+a) \geq 0$

$x \in (-\infty, -a] \cup [a, \infty)$

Ans.

- $|x| > a$, $a \rightarrow$ Positive

$$x \in (-\infty, -a) \cup (a, \infty)$$

OR,

$$x < -a \quad \cup \quad x > a$$

Generalise,

- $|E| \geq a$

  $E \leq -a \quad \cup \quad E \geq a$

- $|E| > a$

  $E < -a \quad \cup \quad E > a$

LEARN

$a \geq$ Positive.

Q. Solve for x :

$$|x + 3| \geq 7.$$

$E = x + 3$

$|E| \geq 7$

$E \leq -7 \quad \cup \quad E \geq 7$

$x + 3 \leq -7 \qquad x + 3 \geq 7$

$\boxed{x \leq -10} \quad \cup \quad \boxed{x \geq 4}$

$x \in (-\infty, -10] \cup [4, \infty)$ Ans.

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Q. Solve for x :

$$\left|\frac{2}{x-4}\right| > 1.$$

$$E = \frac{2}{x-4}$$

$|E| > 1$

$E < -1$ U $E > 1$

$\frac{2}{x-4} < -1$

$\frac{2}{x-4} + 1 < 0$

$\frac{x-2}{x-4} < 0$

$x \in (2,4)$ ✓

$\frac{2}{x-4} > 1$

$\frac{2}{x-4} - 1 > 0$

$\frac{2-x+4}{x-4} > 0$

$\frac{-x+6}{x-4} > 0$

$\frac{x-6}{x-4} < 0$

$x \in (4,6)$ ✓

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$x \in (2,4) \cup (4,6)$ Ans

Q. Solve for x :

$$|-2x + 5| \geq 11.$$

$E = -2x + 5$

$|E| \geq 11$

$E \leq -11$ U $E \geq 11$

$-2x + 5 \leq -11$

$-2x + 16 \leq 0$

$2x - 16 \geq 0$

$x \in [8, \infty)$ ✓✓

$-2x + 5 \geq 11$

$-2x - 6 \geq 0$

$x \in (-\infty, -3]$ ✓✓

final ans

$x \in (-\infty, -3] \cup [8, \infty)$

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Q. Solve for x :

$$\left|\frac{x^2+12}{7x}\right| \geq 1.$$

$E = \frac{x^2+12}{7x}$

$|E| \geq 1$

$E \leq -1$ U $E \geq 1$

$\frac{x^2+12}{7x} \leq -1$

$\frac{x^2+12}{7x} + 1 \leq 0$

$\frac{x^2+7x+12}{7x} \leq 0$

$\frac{(x+4)(x+3)}{7x}$

$x \in (-\infty, -4] \cup [-3, 0)$

$\frac{x^2+12}{7x} \geq 1$

$\frac{x^2+12}{7x} - 1 \geq 0$

$\frac{x^2-7x+12}{7x} \geq 0$

$\frac{(x-4)(x-3)}{7x} \geq 0$

$x \in (0, 3] \cup [4, \infty)$

$$x \in (0,3] \cup [4,\infty) \cup (-\infty,-4] \cup [-3,0)$$

OR,

$$x \in (-\infty,-4] \cup [-3,0) \cup (0,3] \cup [4,\infty)$$

Ans

TYPE 3

→ Neither Type-1 → $|E| \leq a$ OR $|E| < a$

Where making Cases is important

NOR

Type-2

→ $|E| \geq a$ OR $|E| > a$

Q. Solve for x :

$$\frac{|x-1|-2}{x} \leq 0.$$

→ Modulus Inequality

→ opening of mod is Required

Case 1 → $x-1 \geq 0$

$x \in [1, \infty)$

$$\frac{+(x-1)-2}{x} \leq 0$$

$$\frac{x-3}{x} \leq 0$$

$x \in (0, 3]$

Common values leni hai

$AC_1 \in [1, 3]$ ✓

Case 2 ⇒ $x-1 < 0$

$x \in (-\infty, 1)$

$$\frac{-(x-1)-2}{x} \leq 0$$

$$\frac{-x+1-2}{x} \leq 0$$

$$\frac{-x-1}{x} \leq 0$$

$$\frac{x+1}{x} \geq 0$$

$x \in (-\infty, -1] \cup (0, \infty)$

$AC_2 \in (-\infty, -1] \cup (0, 1)$

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final ans →

$AC_1 \cup AC_2$

$x \in (-\infty, -1] \cup (0,1) \cup [1,3]$ ✓

OR

$x \in (-\infty, -1] \cup (0,3]$ Ans

Gyaan →

Cases के अंदर intersection,

Case-Case Union.

final answer

Q. Solve for x :

$$x^2 - |x+2| + x > 0.$$

Case-1 $\Rightarrow$ $x+2 \geq 0$

$x \in [-2, \infty)$

$x^2 - \{+(x+2)\} + x > 0$

$x^2 - x - 2 + x > 0$

$x^2 - 2 > 0$

$(x - \sqrt{2})(x + \sqrt{2}) > 0$

$x \in (-\infty, -\sqrt{2}) \cup (\sqrt{2}, \infty)$

$AC_1 \in [-2, -\sqrt{2}) \cup (\sqrt{2}, \infty)$ ✓

Case-2 $\Rightarrow$ $x+2 < 0$

$x \in (-\infty, -2)$

$x^2 - \{-(x+2)\} + x > 0$

$x^2 + 2x + 2 > 0$

AP_0, CB

$1 > 0$ → valid

$x \in (-\infty, \infty)$

$AC_2 \in (-\infty, -2)$ ✓

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final ans →

$$x \in (-\infty, -2) \cup [-2, -\sqrt{2}) \cup (\sqrt{2}, \infty)$$

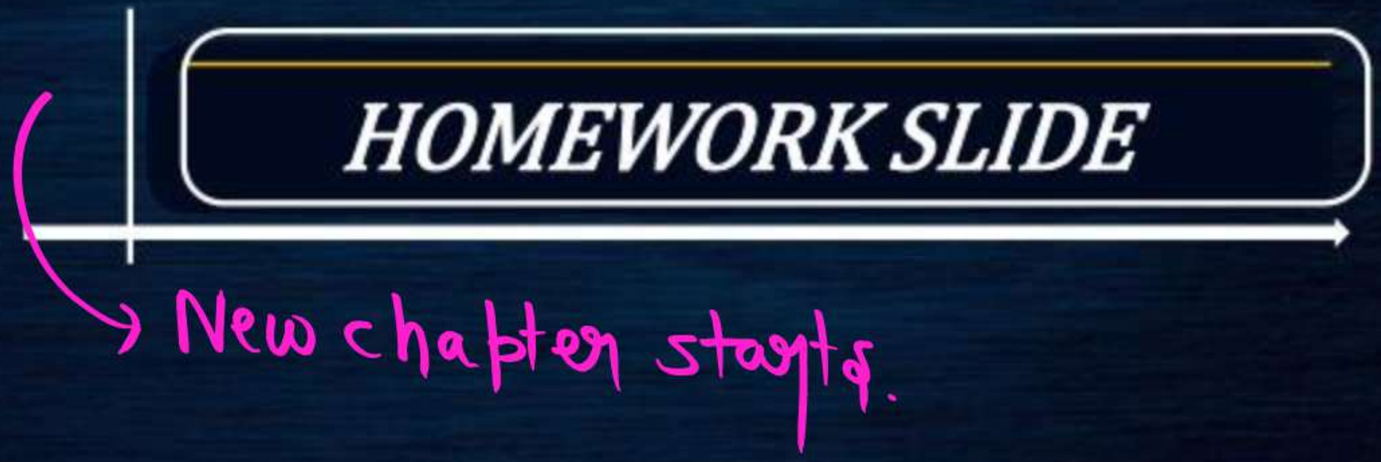
Lecture 1 and 2.

$$\boxed{x \in (-\infty, -\sqrt{2}) \cup (\sqrt{2}, \infty)}$$

Ans

Solve Remaining Questions of all DPPs of BM
(Fighter attitude)

Revise Notes completely using pen and paper.



New chapter starts.

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PW Maharathi

Because Practice makes a Maharathi

You have a TEST !!!

Rewards for Students who attempt all the tests regularly on Sunday/ Monday!

- Top 3 Students in each month will receive a Gift Voucher worth Rs 500.
- Top 20 Students in each month will receive a Gift Voucher worth Rs 200.
- Lucky 20 students in each month will be selected on a random basis who will receive a Gift Voucher worth Rs 100 for attending regular tests.
- All the students attempting regular tests will be eligible for Live Teacher Interaction-Paramarsh(2.0)

MEGA MAHARATHI (Oct-2022 & Feb- 2022)

- Top 5 Students will receive a Gift Voucher worth Rs 1000
- Lucky 100 Students in September 2022 will be given a Gift Voucher worth Rs 500 under MEGA MAHARATHI.

** Eligibility and Rules are covered in the next slide.

DO’s AND DON'Ts TO BECOME A PW MAHARATHI:-

1). You have to attempt all the test occurring between the last Maharathi and till the next one to be eligible for Maharathi at all.

2). Maharathi will be announced only in the batch for the month if more than one test has occurred in that last month. For eg- If only one test occurred in June then that test would be considered in July’s Month Maharathi Results.

3). The combined performance of all the tests would ensure your selection in the toppers prizes.

4). The selection of **Lucky students** will be done using our Random Selection Algorithm and not on the basis of marks, names and past maharathi results, you just have to be eligible by attempting all tests and yes, if you have been selected once it can happen again :D

5). You are eligible in MEGA MAHARATHI if you have given more than 8 TESTS out of the 11 TESTS occurring throughout the batch i.e July-Oct. and not on marks or any other factor.

6). All the tests need to be attempted till the midnight (Tuesday).

7). You will have to attempt all tests genuinely and completely to be eligible for any scheme. We will use our Fraud check algorithms before identifying the award winners.

JBK...JBK...JBK

- TO *FUTURE IIT-IANS*